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BALLISTIC RESEARCH LABORATORIES

REPORT No. 672

Tracking Telescope Fundamentals

JAMES B. EDSON

ABERDEEN PROVING GROUND, MARYLAND

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ABSTRACT

This report assembles a foundation of simple basic information useful in the design and operation of tracking telescopes. Some topics discussed are:

Appearance of the missile compared to planets; conditions for optimum missile visibility; telescope size requirements; effects of the atmosphere; station site selection; tracking and photographic techniques.

Construction and installation at White Sands Proving Ground of the first tracking telescope is described.

INTRODUCTION

Methods and Objectives.

This report deals with the observation by large astronomical telescopes of long range missiles in and above the Atmosphere.[1]

The purpose of such observations is to secure one or more of the following types of information:

1. Aspect of the Missile. By observing the missile as an image of finite area to determine the nature of yaw oscillations, rotation, structural failure of parts, the manner of departure of items deliberately thrown from the missile, and in general the details of all externally visible events in the vicinity of the missile.

2. Optical Telemetering. By observing special optical signals sent from the missile to measure quantitatively various conditions in the missile and its environment.

3. Location of the missile in space. By means of graduated circles or other methods to locate the missile at great distances and/or with high accuracy.

4. Fire Control. To gather experience against the time when such equipment may be used in connection with ground or other methods of fire control.

Progress to Date.

Since its inception this project has assembled a foundation of simple basic theory and has upon that basis begun the design, development and testing of a series of experimental instruments. The first of these instruments, with their auxiliary equipment, have been in operation at White Sands Proving Ground.

Experience with these is yielding satisfactory information concerning the design, construction, and use in the field of such equipment.

SOME BASIC CONSIDERATIONS

Appearance of the Missile.

1. Brightness of the Missile. If the missile is coated with a good coat of white matt finish paint, it will reflect according to Lambert's law. Then from any longitudinal differential strip of the cylindrical missile body the observer receives an amount of light:

$$\frac{Brl}{D^2} \cos\omega \cos(\emptyset - \omega)\, d\omega$$

where

     B = intensity of sunlight at the missile

     r = radius of the missile

     l = length of the missile

     D = Distance from observer to missile

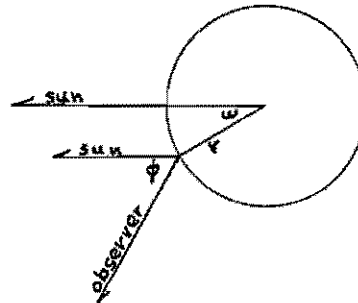and the angles $\emptyset$ and ω are defined in Figure 1.

Figure 1

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[1] L. A. Delsasso, L. G. deBey, D. Reuyl, "Full Scale Free-Flight Ballistic Measurements of Guided Missiles", BRL Report No. 660, January 1948.

The light received by the observer from the projectile is:

$$L = \frac{Br1}{D^2} \int_{\theta-\pi/2}^{\pi/2} \cos\omega \cos(\emptyset - \omega)\, d\omega$$

$$= \frac{Br1}{2D^2} \left[(\pi - \emptyset) \cos\emptyset + \sin\emptyset \right]$$

With a telescope showing the projectile as a bright line of no appreciable width, the total light will be distributed along the line, and the brightness will thereafter vary independently of 1 and inversely as D instead of D^2.

Figure 2 shows L as a function of distance along the ground measured from a point under the maximum ordinate H and along a line perpendicular to the trajectory plane. Both the $1/D^2$ and $1/D$ cases are plotted. These curves indicate that where two stations are involved one station should be located about H/2 to sunward of the orbit plane, and the other can be located at about 3H/2, giving acceptable triangulation conditions.

2. Brightness of Missile Compared to That of Planets. Assuming the projectile to be in the full solar beam, the brightness of the projectile compared to any planet considered on a point source basis can be approximately estimated by comparisons of solar distance, albedo, and angular size of illuminated area. If

S_R = solar distance of rocket 93×10^6 mi.

S_P = solar distance of planet

A_R = Albedo of rocket assumed ≈ 0.8

A_P = Albedo of planet

G_R = Illuminated angular area of rocket

G_P = Illuminated angular area of planet

L_R = Brightness of rocket as point source

L_P = Brightness of planet as point source

Then

$$\frac{L_R}{L_P} \cong \frac{G_R}{G_P} \frac{A_R}{A_P} \left(\frac{S_P}{S_R} \right)^2$$

FIG. 2

RELATIVE STELLAR BRIGHTNESS OF PROJECTILE AT SUNRISE OR SUNSET
AS A FUNCTION OF HORIZONTAL DISTANCE FROM THE GROUND
PROJECTION OF THE TRAJECTORY IN UNITS OF PROJECTILE ALTITUDE
AXIS OF PROJECTILE ASSUMED HORIZONTAL

RELATIVE BRIGHTNESS

HORIZONTAL DISTANCE

Estimates along these lines indicate that a projectile 8 inches by 8 feet seen at 285,000 feet under full illumination would be about as bright as the planet Mars, both being seen as points. However, in a telescope capable of showing the projectile as an extended area, the projectile would show a surface brightness about 9.3 times that of Mars and about 0.66 times that of Venus. At the optimum brightness angles of $\emptyset = 63°$ the stellar brightness of such a projectile would be about 0.5 that of Mars. For a projectile of 24 inch diameter and 30 ft. length the stellar brightness would be about the same as Venus and about ten times that of Mars. At the optimum brightness angle such a projectile would have a brightness about 0.5 Venus and 5.0 Mars.

3. Optimum Telescope Size for Visual Observation. The visibility of the target depends upon the objective diameter and the magnification of the observing telescope. For a point source the contrast of the source against the sky background will increase roughly with the square of the objective diameter. When the telescope magnification is sufficient to show the projectile as a line of no sensible width, the image contrast will increase proportionally to the objective diameter. When finally the telescope reveals the projectile as an object of visible width as well as length, the contrast of the image against the sky can no longer be increased. Beyond this point further increase in objective diameter is, for tracking purposes, not so important.

Thus the two quantities most significant in telescope choice are the objective diameters where (1) length is resolved, and (2) where width is resolved.

The resolving power of a telescope is approximated by

$$\emptyset = \frac{1.17'}{h}$$

where $\emptyset$ = minimum resolvable angle in min. of arc

h = objective radius in min.,

Then for a projectile 1 long and 2r wide at a distance D the critical objective radius h is

$$h_{length} = \frac{1.17}{3438} \frac{D}{1} = 3.4 \times 10^{-4} \frac{D}{1}$$

$$h_{width} = \frac{1.17}{3438} \frac{D}{2r} = 3.4 \times 10^{-4} \frac{D}{2r}$$

TABLE I

Minimum telescope objective diameter to resolve a given
length at a slant range of 300,000 feet.

Length (feet)	Distance (divided by) Length	Objective Diameter (millimeters)
5	60,000	41.2
10	30,000	20.4
15	20,000	13.6
20	15,000	10.0
30	10,000	6.8
40	7,500	5.2
50	6,000	4.1
60	5,000	3.4

The variation of image brightness with telescope objective diameter is for the case of a projectile of length 10 times its diameter shown in Figure 3.

Optimum telescope size for visual observation and tracking will also be affected by such factors as:

(a) Increasing instrument cost and weight as telescope size increases.

(b) Decreasing angular field of view and increasing magnification as telescope size increases.

(c) Sky conditions and duration of twilight at firing range.

4. Brightness of Sky Background. The conspicuousness of the projectile in the telescope field will in practice depend upon the ratio of image brightness to sky brightness. Sky brightness at time of observation will thus set a lower limit upon the size of the telescope required, and may even prevent observation of the projectile entirely.

When two surfaces of different brightness have a common boundary, the eye can just perceive a difference of brightness between them of about 2%. A difference of 10% is easily perceived, and when one surface is twice as bright as another the difference is conspicuous. For this study we will assume a brightness ratio of 2 for image/sky as the minimum required for satisfactory tracking. Since the brightness of clear daylight sky is about 0.4cd/cm^2 and the brightness of the "full" projectile will run as high as 2.73cd/cm^2 it is plain that with a telescope capable of resolving the projectile into an area, the projectile will be visible in daylight sky. As a basis of comparison, the following area brightnesses are given:

Moon (full)	0.2 candles/cm^2
Mars (full)	$0.20 \text{ candles/cm}^2$
Venus (full)	$3.81 \text{ candles/cm}^2$
Projectile (full)	$2.73 \text{ candles/cm}^2$
Day Sky (full)	0.4 candles/cm^2

The maximum surface brightness of the projectile will be about 10 times that of the Moon. Since the Moon is easily visible in the day sky, the projectile, seen as an area, will be conspicuous.
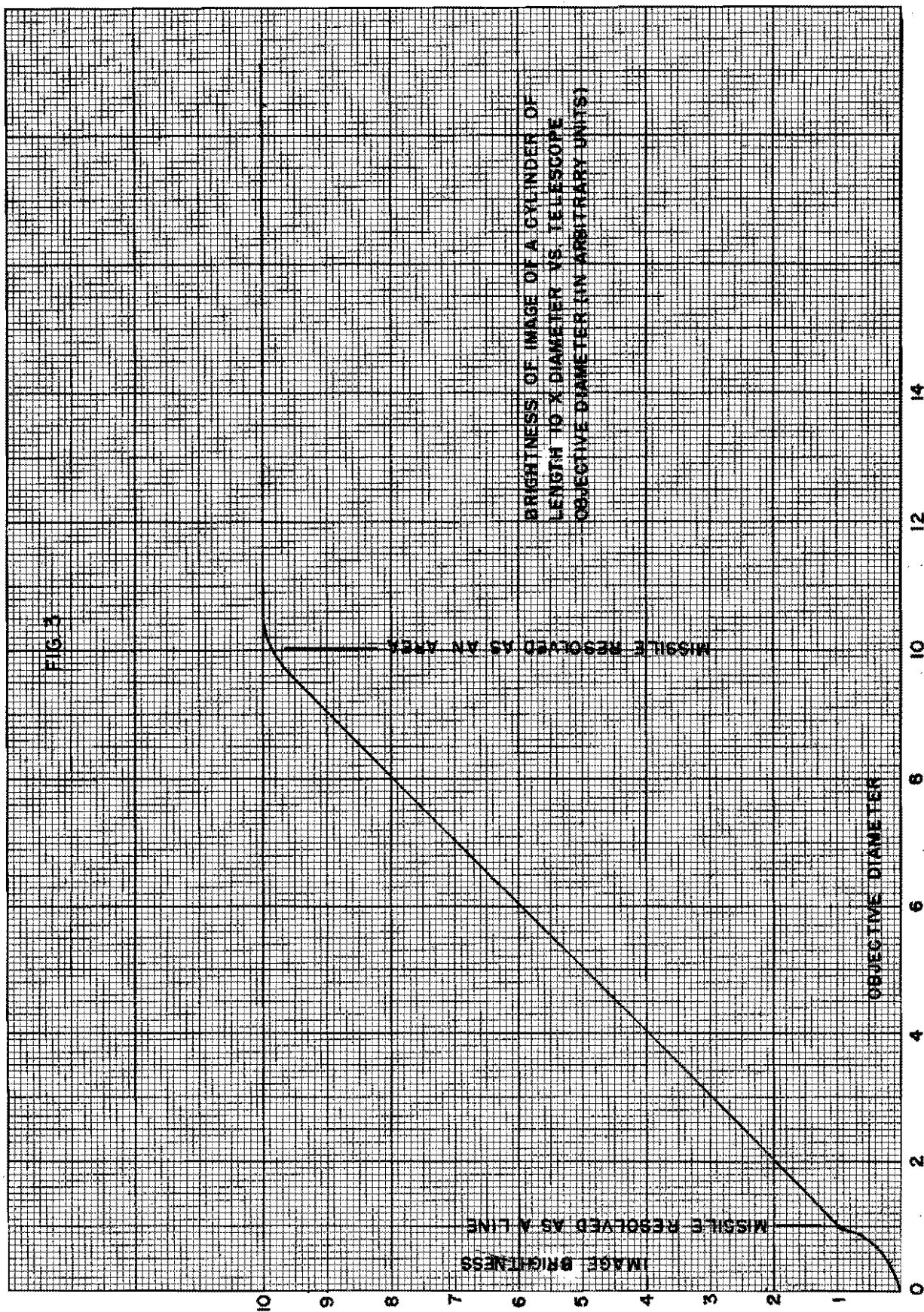
Recalling

$$L = \frac{Brl}{D^2} \left[(\pi - \emptyset) \cos \emptyset + \sin \emptyset \right]$$

we have, setting D = 1 and $\emptyset$ = 0, the total brightness of a 3' x 30' projectile as Brl = 176,000 candles with an intensity at unit distance of 176,000 lumens per square foot = 1.90×10^6 lumens per square meter at a distance of 3×10^5 ft., the intensity of the "full" projectile is

$$\frac{1.76 \times 10^5}{9 \times 10^{10}} = \frac{1.76}{9} \times 10^{-5} = 1.95 \times 10^{-6} \text{ ft. candles or}$$

$$\frac{1.90 \times 10^6}{9 \times 10^{10}} = 2.1 \times 10^{-5} \text{ lumens per meter}^2$$

FIG. 3

BRIGHTNESS OF IMAGE OF A CYLINDER OF
LENGTH 10 X DIAMETER VS. TELESCOPE
OBJECTIVE DIAMETER (IN ARBITRARY UNITS)

MISSILE RESOLVED AS AN AREA

MISSILE RESOLVED AS A LINE

IMAGE BRIGHTNESS

OBJECTIVE DIAMETER

At the optimum angle of view, 51.50, or 0.895 radians we have:

$$\frac{Brl}{2}\left[(\pi - \emptyset)\sin\emptyset\cos\emptyset\right] = \frac{2.7}{2} \times 1.5 \times 30 \times 30.48^2 \times 1.70$$

$$= 96,000 \text{ candles} = 1.04 \times 10^6 \text{ lumens/m}^2 \text{ at 1 ft.}$$

At the optimum angle of view and 300,000 ft. range the total brightness of the "phased" projectile will be

$$1.14 \times 10^{-5} \text{ lumens/meter}^2$$

Knowing that the intensity of a star of 1st magnitude is

$$E_1 = 8.3 \times 10^{-7} \text{ lumens/meter}^2, \text{ and}$$

$$E_m = E_1 (1/2.5)^{m-1} \text{ we have}$$

$$\frac{E_m}{E_1} = \frac{1.04}{8.3} \times 10^2 = 12.5 = (1/2.5)^{m-1} = 2.5^{1-m}$$

and $m = -1.86$, stellar magnitude of the projectile at optimum angle and 300,000 ft. range.

A table for stellar magnitude of projectile at optimum angle and various ranges is included.

TABLE II

Stellar magnitude of a projectile 3 ft. diameter, 30 ft. long, Albedo = 0.8, phase angle = 51.50 = 0.895 radians, at various ranges. Candles power of projectile under these conditions estimated at 96,000 cp. = 1.01 × 10⁶ lumens/meter² at one foot distance.

Range (ft)	Stellar Magnitude	Range (ft)	Stellar Magnitude	Range (ft)	Stellar Magnitude
1×10^5	-4.26	6×10^5	-0.36	20×10^5	+2.16
2×10^5	-3.76	8×10^5	+0.27	30×10^5	+3.15
3×10^5	-1.88	9×10^5	+0.53	50×10^5	+4.28
4×10^5	-1.25	10×10^5	+0.75	100×10^5	+5.78
5×10^5	-0.76				

Effects of the Atmosphere.

1. Sky Background. The visibility of the projectile against the normal day sky has been discussed. The relative brightness of the background and the missile will be affected by the following:

(a) Angular height of the missile

(b) Angular distance from the sun

(c) Angular height of the sun

(d) Haze

(e) Elevation of the observing station

(f) Use of color filters

(g) Use of a polarizing filter

(h) Stray and scattered light in the optical system

When the angular height of the missile is small, the missile is seen against a great depth of hazy atmosphere near the horizon. The background then will be several times that of the sky above 30° elevation. As the missile rises the background illumination falls off, at first rapidly, and then more slowly.

The change above 30° due to this "bright horizon" effect is usually relatively small. As a result of this horizon haze the rising missile at first becomes steadily brighter relative to the sky. In the most unfavorable cases where the projectile is poorly painted and/or presents a shadowed side to the camera, it may first be seen as a dark silhouette. As it rises it will slowly fade and vanish and then reappear as a bright object against the sky. This was actually observed at several of the White Sands firings.

The effect of angular distance from the sun may be very great or quite small, depending upon the nature and number of particles in the air. At White Sands, particularly at altitudes of 5000 feet or more, the white glare around the sun was seen usually to be very faint. Atmospheric sun glare will not be a major factor in observations made there.

The angular height of the sun controls many variables important to the optical observation of long range missiles. Sky brightness is among these. The brightness of a clear sky is greatest at noon. It decreases with the solar distance from the horizon, at first slowly, then with increasing rapidity, until near and after sunset the percentage rate of change becomes very large. For the variation during the day, see Hardy & Perrine "Principles of Optics", page 161, Figure 73. Figure 4 gives a graphic representation of the change near sunset. In considering effects of solar position, the changes in illumination of the projectile must always be borne in mind.

Haze, when present, has a very bad effect on missile observations. Morning and evening haze, caused by radiation to space, are usually relatively shallow and many of the higher localities in a mountainous region like White Sands are above the haze layer. These higher localities are for this and other reasons to be preferred as tracking telescope sites.

Some of the helpful effects of a high altitude observing station have been mentioned above. Even with perceptible haze the sky is substantially darker at 8,000 ft. than at 5,000 ft. This effect is particularly marked in the part of the sky above 10° to 15°. The horizon itself can be quite bright with low-lying haze, even at high altitudes.

Because there is less air over higher stations, the atmospheric unsteadiness called "astronomical seeing" is usually less, and a sharper image results.

Since the light of a clear sky is due mostly to multiple Rayleigh scattering, its brightness varies roughly inversely as the fourth power of the wave length considered. With a white painted projectile, the use of yellow or red filters will accordingly increase the missile-to-background brightness ratio. However, such filters remove part of the light from the image and thus increase exposure time. There is an optimum wave length at which the filter cut should be made. This optimum depends on the instrument size and other factors.
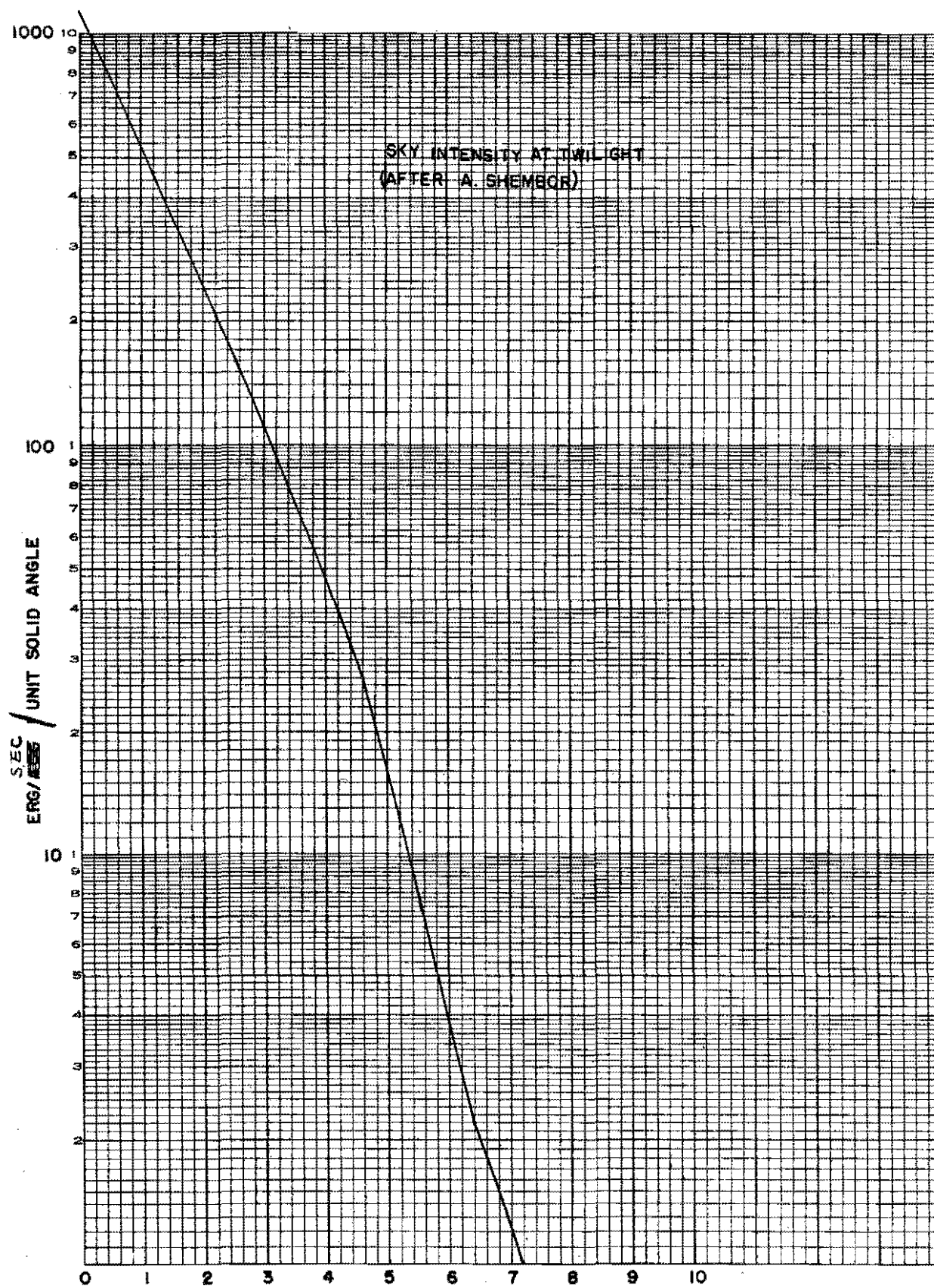
FIG. 4   SOLAR DEPRESSION DEGREES

Station Selection.

1. Astronomical Seeing. It is important to select an observing site where the definition - or "seeing" - is of usable quality nearly all of the time. Factors of importance in searching for a station with good seeing are:

(a) Altitude. The station should be at an elevation of 5000 ft. or higher if possible with trees, shrubbery, or at least grass. Bare ground produces temperature extremes which greatly disturb the air over it.

(b) Shape of the Terrain. Modern opinion prefers a plateau of some extent rather than a sharp peak because strong turbulent up and down drafts may disturb the air about a peak. However, a valley or depression atop a plateau should be avoided. The station should be located on a knoll, preferably the highest in the vicinity, and the site should be checked for seeing by observation.

2. Geometry of Illumination. According to the results in "Basic Considerations", above, satisfactory sites for study of the higher parts of the trajectory may advantageously be located on a horizontal line normal to the trajectory, and passing under the maximum ordinate at a distance from the ground projection of the trajectory equal to between 1/2 and 3/2 of the maximum ordinate. The assumption there was for a north-south trajectory and firings near sunrise or sunset. For firings nearer noon, the optimum locations would be somewhat farther from the ground projection of the trajectory. The theory in "Basic Considerations", can be easily modified to give quantitative results for any time of firing desired. .

For a study of the phenomena during burning or details of behavior during reentry into the atmosphere, special locations nearer to the launching site or impact area may be of advantage.

INSTRUMENTATION AND TECHNIQUES

Tracking the Missile.

With the tracking telescope, the maximum angular tracking rate is low, about 5° per second. The angular target accelerations will not exceed one or two degrees per second^2 at the utmost. However, the large image scale makes comparatively rigid demands upon tracking accuracy. In order to hold the target in the camera field of even the smallest tracking telescope, the tracker must maintain an angular error less than four minutes of arc. The tracking rate error also must be so small that the image does not move appreciably on the film during the exposure time. This means that even in the smallest tracking telescope the picture will be appreciably blurred when the tracking rate error reaches one minute of arc per second of time.

The necessary conditions to achieve this accuracy are:

1. Sufficient power in the tracker's visual telescope to show errors.

2. Sufficient skill in the observer to note and correct errors as they occur.

3. Sufficient precision in the mechanical drive and controls of the mount to permit correction of errors.

To keep the image accurately centered, the observer must be able to resolve in his telescope an angle equal to, say, 5% of the camera field angle. For a field of 8 minutes, as in the smallest tracking telescope, this requires a magnification which will reveal an angle of 0.4 minutes. The resolving power

of the eyes is about one minute. Therefore a magnification of 2X will be sufficient to center the image in this case.

A trained operator can comfortably make corrections at the rate of, say, one per second. The motion of a moving object can be comfortably observed within intervals of one second when the angular velocity of the object in the eye is, say, five minutes of arc/sec of time. This, ideally, allows a velocity estimate error of $\pm 20\%$ for each second. For the above maximum tracking rate error of one minute of arc per second of time, a minimum magnification of 5X is required in the visual observer's telescope.

Finally, the most severe demands on both magnification and the resolving power of the tracker's telescope are made by considerations of image size and visibility. This has been discussed previously under "Optimum Telescope Size for Visual Observation".

To review it here, an image for optimum visibility must appear to the observer as a perceptible area. The A4 missile at 60 miles has an approximate angular diameter of 5 seconds of arc. To make an appreciable area on the retina it must appear, say, 2 minutes wide. This requires a magnification of approximately 25. It is evident that the 20X binoculars represent a minimum satisfactory power for the observation of the image as an area in this case.

As the magnification of the tracker's telescope is increased, the size of his field is diminished, until at some point difficulty of picking up the projectile and the danger of losing it become serious.

With the 2° field on the 20X binoculars, pickup has been found very difficult unless an open gunsight is first used as a finder. However, the chance of losing the target from the field is negligible, provided the target is large enough to be clearly seen.

Cinephotography of the Missile.

1. Camera. The tracking telescope records the image of the missile by making a series of photographs in rapid succession. For the first instrument this has been done by means of a 35mm Eyemo motion picture camera, at frame speeds of about 16 frames per second. The lens is removed and the image from the telescope amplifying lens is focused on the film.

2. Exposure Time. A necessary condition for high percentage of sharp images is that the exposure time should be short enough to prevent blur due to tracking rate error. For good tracking with the present equipment, an exposure time of about 1/60 of a second is sufficiently short.

3. Image Scale. In order to obtain sufficient image density in the required short exposure time it is necessary to limit the focal ratio, filter factor, and image scale.

If

t = exposure time
M = image scale
F = filter factor
B = sky brightness
D = diameter of telescope objective
K = a constant depending on film sensitivity

Then

$$t = \frac{KM^2 F}{BD^2}$$

It was found at White Sands that the following set of values gave satisfactory results with Super XX panchromatic film:

$$t = 1/60 \text{ secs.} \qquad\qquad B = 0.4 \text{ candles/cm}^2$$

$$M = 2.5 \text{ mm/minute of arc} \qquad D = 4.5 \text{ inches}$$

$$F = 2 \quad \text{(K2 filter)}$$

Hence

$$K = \frac{BD^2T}{M^2F} = \frac{0.4 \times 4.5^2 \times 1/60}{2.5^2 \times 2}$$

$$K = 0.11 \text{ for Super XX film}$$

For other conditions and instruments, if the Super XX film is used, the maximum image scale is then given by

$$M = \frac{tBD^2}{KF}$$

Finally as instrument size increases, an upper limit to the useful image scale is set by the astronomical seeing. Experience with the Lowell 24" refractor indicates that this upper limit lies in the vicinity of 12 mm/minute of arc for the conditions at Flagstaff, Arizona. Conditions in pine clad mountains near White Sands should be similar to those at Flagstaff. Observations at various station locations at White Sands Proving Ground show that Flagstaff experience is acceptable as a preliminary guide to White Sands expectations.

4. Choice of Photographic Emulsion. Since the vital importance of short exposure time has been pointed out it might at first be thought that the "fastest" film would be best for tracking telescope use. Such a conclusion is not correct. The "speed" ratings commonly used for photographic emulsions involve in one way or another the inertia or the threshold of the emulsions. In photography with the telescopes, especially against the day sky, inertia plays a different role. A considerable inertia may actually be desirable because inertia is associated with high contrast.

Let us consider the effects of various emulsion characteristics upon the exposure time necessary to yield an image of given photographic quality. This exposure time is not an ideal parameter for the measurement of image quality, because image quality is not inversely proportional to exposure time. Nevertheless, the shortest exposure time can be expected to yield the best optical image. We saw above that $t = \dfrac{KM^2F}{BD^2}$ where K is a constant depending upon film sensitivity. For our own purposes let us define film sensitivity

$$K = \frac{C \, \Delta(It)}{\Delta D}$$

where

$$\Delta D = \text{density difference}$$

$$\Delta(It) = \text{exposure difference}$$

$$C_1 = \text{a constant}$$

Now it is also clear that, for a given angular resolution on the photographed image, image scale can be inversely proportional to the linear resolution of the emulsion. This gives us the relation between image scale and plate resolution $M = C_2/R$ where $R =$ linear plate resolution. Substituting

$$t = \frac{C_3 F}{BD^2 R^2 \frac{\Delta D}{\Delta(It)}} \quad \text{where } C_3 = C_1 C_2$$

Here we have a general expression involving t and emulsion characteristics. The plate resolution R may be approximated directly by tests with fan or other suitable test objects, or it may be resolved into its components. One significant analysis involves graininess and turbidity:

$$R = \frac{C_4}{L \Delta K}$$

where $L =$ probable error of turbidity distribution function

$\Delta K =$ absolute graininess from photometer fluctuations

$$t = \frac{C_5 F L^2 (\Delta K)^2}{BD^2 \frac{\Delta D}{\Delta(It)}} \qquad C_5 = C_3 C_4$$

So a measure of plate quality E for our purposes is:

$$E = \frac{\frac{\Delta D}{\Delta(It)}}{L^2 (\Delta K)^2}$$

This equation shows the importance of minimizing turbidity and graininess. It should be noted that in practice these two quantities are related, turbidity tending to increase as graininess decreases.

Since ΔK increases with the conventional "speed" rating of emulsions, it is seen that the conventionally "fastest" film may not be best for tracking telescope use. It is also well known that ΔK can be reduced by various "fine grain" developers and developing techniques. Such techniques frequently involve a large decrease in $\Delta D/\Delta(It)$, so that indiscriminate use of "fine grain" development is not recommended. The turbidity L varies with the wave length of the light used. It can be reduced by limiting the photographic image to a layer very near the emulsion surface. This once again tends to reduce $\Delta D/\Delta(It)$, and may also cause an increase in ΔK.

Experience with planetary images indicates that, for satisfactory results the sky background should be between densitites 0.2 and 1.0, and the minimum density difference between image edge and background should be about 0.6. The maximum useful image-to-background contrast is represented by a density difference of about 1.2. That is: $0.6 \leqslant \Delta D \leqslant 1.2$. This leads us to a closer consideration of the expression $\Delta D/\Delta(It)$.

The $\Delta(It)$ means the exposure <u>difference</u> between sky background and missile image. It is the additional exposure beyond sky background exposure. The sky background exposure can be thought of as used up in overcoming inertia of the photographic emulsion and building up density to a point where the emulsion's

dD/d(It) approaches a maximum. If sky light is not sufficiently intense to accomplish this purpose, the field can be given a uniform artificial illumination to make up the deficit. Field or sky illumination in correct amount can thus add 30% to 60% to the effective speed $\Delta D/\Delta$ It of an emulsion. It is especially effective in speeding up the slower, contrastier, finer grained emulsions. Supplementary exposures are always helpful for night photography where the sky background is absent.

Since ΔD is set at 0.6, we can get an expression for I as follows:

$$t = \frac{CFL^2 (\Delta K)^2 \Delta(I)t}{BD^2 \times 0.6}$$

The t on both sides refers to the same exposure time; being equal, they can be cancelled leaving:

$$I = \frac{0.6\ BD^2}{C_5 FL^2 (\Delta K)^2}$$

Hence, for practical purposes, there exists a minimum useful projectile surface brightness which depends on emulsion and instrument characteristics. It is noted that this result is forced by the arbitrary choice of ΔD. It assumes that the background intensity is controllable to secure optimum results with any given emulsion. It has no connection with the phenomenon noted when a poorly painted and/or illuminated projectile leaves the ground as a "dark" object, fades to match the darkening sky as it ascends, and finally reappears as a "bright" object at higher elevation angles. The arbitrary setting of ΔD can be questioned on theoretical grounds, because the necessary ΔD depends in part on ΔK. Were no graininess present, the image contrast could be stepped up by high contrast copying. However, such techniques require great care when used on images intended for quantitative measurement, and there are other limitations.

For the present, available data show shellburst panchromatic film to be the best available film. In the future, further study and development may lead to emulsions superior to those now available.

FIELDS OF USE

Aspect of the Missile.

The tracking telescope can determine the attitude of the missiles, the orientations of its various axes in space. It can also report any externally visible incidents during the missile flight. Collapse, breaking off, or large displacements of structural parts are observed without need to predict them. The same is true of deliberate ejection of parts from the missile; for example, warheads and instrument boxes. Such parts can, if desirable, be tracked down to the ground, thus locating them for recovery.

Later in the development of long range rockets there may occur at high altitudes in and above the atmosphere an increasing number of incidents observable to special advantage with tracking telescopes. Among these are the functioning of the successive stages of step rockets, the working of steering jets, of check jets to slow the fall, of warheads, and various other components.

Location of the Missile in Space.

Because of its ability to track and record the missile at great ranges, the tracking telescope may serve well for the measurement of position also.

At the present time no attempt has been made to equip the instrument with precision circles, but moderately accurate positional data can be obtained with its counters. For more precise reduction of attitude data the time records will serve to establish the tracking telescope orientation by means of other instrumentation.

DEVELOPMENT OF THE INSTRUMENTATION

First Trials.

The preceding discussion showed the optical possibilities to be promising but a crucial point requiring empirical determination was the tracking accuracy. To settle this point, an experimental tracking telescope was constructed from a modified Mount, Machine Gun, Multiple, M45. Upon this mount were installed a pair of 20X Japanese binoculars for tracking, plus a 4.5 inch refracting telescope of 60 inch focal length with amplifying lens and Eyemo motion picture camera for photography.

Tracking and photography of aircraft was successfully carried out with this equipment, proving that with a properly trained tracker the required tracking accuracy could be achieved even with the relatively crude equipment used. The photographic performance of the instrument verified the predicitions made for it.

"Little Brighteyes".

A series of elaborations and improvements evolved the instrument to the stage shown in Figure 5. The developments included a more rigid optical mounting bracket, a chair and telescope for a visual observer, an interphone system for the crew, a camera and dials for recording azimuth and elevation, and increased storage battery capacity. The outfit was given the nickname "Little Brighteyes".

Meanwhile, a specially equipped electronics and maintenance shop truck was prepared. This truck contained radio transmission and reception facilities, electronic timing equipment, a phonograph recorder for the visual observer's report, and all other equipment deemed necessary for operation of the tracking telescope station in the field. Included were five sets of German and Japanese aircraft spotting binoculars.

A special radio-equipped weapons carrier named "Brighteyes Explorer" was prepared for the task of searching for station sites in the desert and mountainous country around White Sands.

Larger Instruments.

After tests at Aberdeen, all of this equipment was moved in convoy to White Sands. Figure 6 shows the first tracking telescope station in action at White Sands Proving Ground. The instrument was later modified by substitution of photographic for visual observation on the right hand telescope, as shown in Figure 7.

Meanwhile work has been in progress on larger instruments. Two of these carry 10 inch reflecting photographic telescopes and are built on the same type of M-45 machine gun mount used for "Little Brighteyes". The third instrument includes a 16 inch reflecting photographic telescope on a 90mm gun M-2. One of the 10 inch instruments is now in operation and the 16 inch telescope is going through initial tests at its site on Mule Peak, near White Sands Proving Ground.
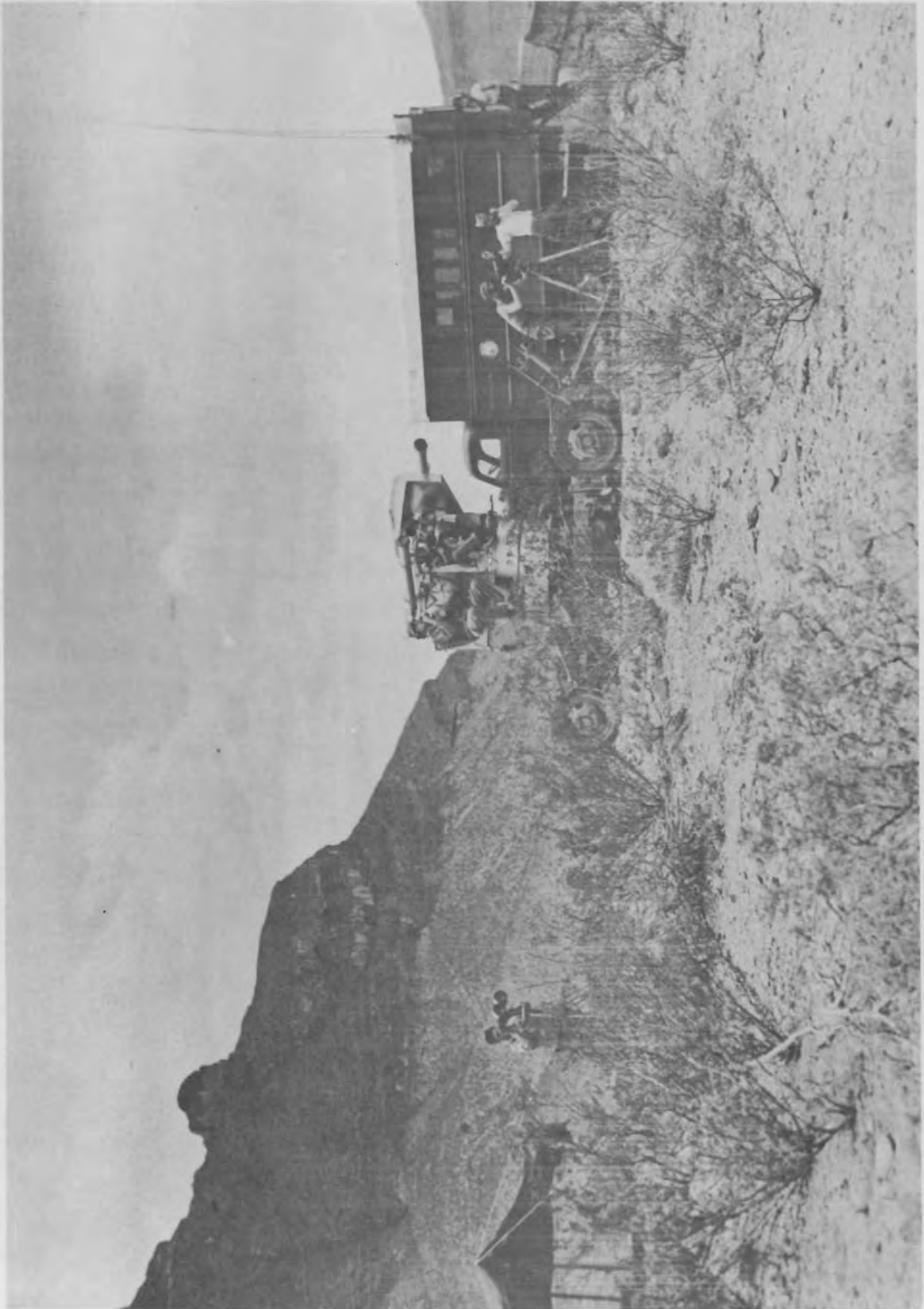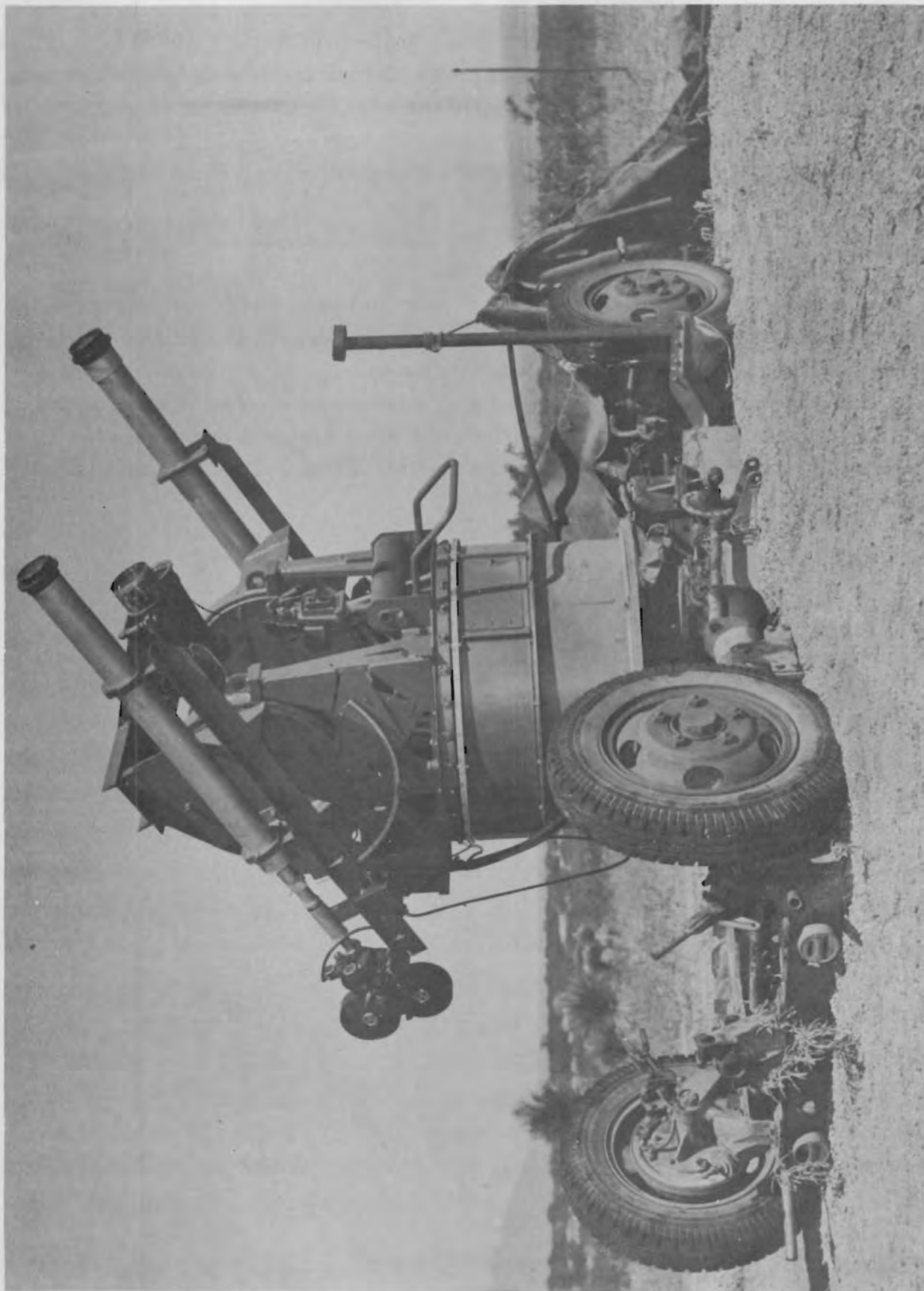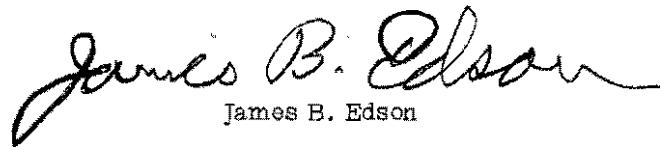
Figure 5

Figure 6

Figure 7

Field Tests.

The first station site was selected on a knoll about 1/4 mile NW of Station F at White Sands and about 9 miles NW of the firing point and established on July 7, 1946. The next day the station went into action to record the firing of an A-4 missile. The missile was tracked and photographed until it disappeared behind clouds shortly after burn-out.

The five sets of AA spotting binoculars and other auxiliary instruments provided a valuable opportunity for the crew members to obtain tracking practice on the missile.

From that time until the firing of August 23, 1946, the technical equipment was serviced and improved and tracking practice with aircraft was carried out.

Further work on this and other instruments has been carried out by others. This includes the modification of the first instrument into a twin photographic telescope as well as the extensive tests of the first 10 inch reflector and the initial test of the 16 inch instrument.
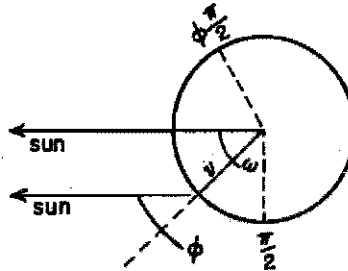
Also work is in progress on the synchronization of the instruments to permit the determination of the direction of the missile axis in space. A special optical stereocomparator is being designed to obtain these data from measurements of the films taken at the tracking telescope stations. Tests with a bread board model have been successful.

James B. Edson

APPENDIX

Problem of Maximum Target Visibility

Assume Lambert's Law of reflection. Then from any differential strip of rocket body the observer receives an amount of light:

$$\frac{Br}{D} \cos\omega \cos(\emptyset - \omega) \, d\omega$$

where

$\qquad$ B = intensity of light reflected from surface at $\omega = 0$
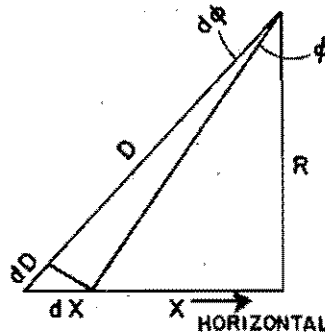
And the total light received from the projectile is:

$$L = \frac{Br}{D} \int_{\emptyset - \pi/2}^{\pi/2} \cos\omega \cos(\emptyset - \omega) \, d\omega$$

$$= \frac{Br}{D} \int_{\emptyset - \pi/2}^{\pi/2} \cos\omega \cos^2\omega \, d\omega + \int_{\emptyset - \pi/2}^{\pi/2} \sin\emptyset \sin\omega \cos\omega \, d\omega$$

$$= \frac{Br}{D} \left[\cos\emptyset \left(\tfrac{1}{2}\omega - \tfrac{1}{4}\sin 2\omega \right) + \sin\emptyset \left(\tfrac{1}{2}\sin^2\omega \right) \right]_{\emptyset - \pi/2}^{\pi/2}$$

$$= \frac{Br}{D} \left\{ \cos\emptyset \left[\frac{\pi - \emptyset}{2} + \tfrac{1}{4}\sin\pi - \tfrac{1}{4}\sin(2\emptyset - \pi) \right] \right.$$

$$\left. + \frac{\sin\emptyset}{2} \sin^2 \pi/2 - \frac{\sin\emptyset}{2} \sin^2(\emptyset - \pi/2) \right.$$

$$= \frac{Br}{D} \left[\cos\emptyset \left\{ \frac{\pi - \emptyset}{2} + \tfrac{1}{2}\sin\emptyset \cos\emptyset \right\} + \frac{\sin\emptyset}{2}(1 - \sin^2(\emptyset - \pi/2)) \right]$$

$$= \frac{Br}{2D} \left[(\pi - \emptyset) \cos\emptyset + \sin\emptyset \cos^2\emptyset - \sin\emptyset \sin^2\emptyset \right]$$

$$L = \frac{Br}{2D} \left[(\pi - \emptyset) \cos \emptyset + \sin \emptyset \right]$$

$$\frac{dL}{d\emptyset} = \frac{Br}{2D} \left[- (\pi - \emptyset) \sin \emptyset - \cos \emptyset + \cos \emptyset \right]$$

$$= \frac{-Br}{2D} \left[(\pi - \emptyset) \sin \emptyset \right]$$

$$\frac{dL}{dD} = \frac{-Br}{2D^2} \left[(\pi - \emptyset) \cos \emptyset + \sin \emptyset \right]$$

From the diagram:

$$\frac{d\emptyset}{dX} = \frac{- \sin \emptyset}{D} \qquad\qquad \frac{dD}{dX} = \cos \emptyset$$

Now at maximum target visibility, $\frac{dL}{dX} = 0$, and

$$\frac{dL}{dX} = \frac{\partial L}{\partial \emptyset} \frac{d\emptyset}{dX} + \frac{\partial L}{\partial D} \frac{dD}{dX}$$

or, at sunrise or sunset:

$$\frac{dL}{dX} = \frac{-Br}{2D} (\pi - \emptyset) \sin \emptyset - \frac{\sin \emptyset}{D} - \frac{Br}{2D^2} \left[(\pi - \emptyset) \cos \emptyset + \sin \emptyset \right] \cos \emptyset$$

$$= \frac{Br}{2D^2} (\pi - \emptyset) \sin^2 \emptyset \frac{-Br}{2D^2} \left[(\pi - \emptyset) \cos^2 \emptyset + \sin \emptyset \cos \emptyset \right]$$

$$= \frac{Br}{2D^2} \left[(\pi - \emptyset) (\sin^2 \emptyset - \cos^2 \emptyset) - \sin \emptyset \cos \emptyset \right]$$

For maximum visibility at sunrise or sunset

$$(\pi - \emptyset)(\sin^2 \emptyset - \cos^2 \emptyset) = \sin \emptyset \cos \emptyset$$

$$(\pi - \emptyset)(\tan \emptyset - \cot \emptyset) = 1$$

$$\tan \emptyset - \cot \emptyset = 1/\pi - \emptyset$$

Computation gives the value of $\emptyset$ for optimum projectile visibility at sunrise or sunset as $\emptyset \simeq 51.5°$.

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